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On nonexistence of magnetic charge in pure Yang-Mills theories

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Abstract: We prove that magnetic charge does not exist as a physical observable on the physical Hilbert space of the pure SU(2) gauge theory. The abelian magnetic monopoles seen in lattice simulations are then interpreted as artifacts of gauge fixing. The apparent physical scaling properties of the monopole density in the continuum limit observed on the lattice are attributed to the correct scaling properties of physical objects - magnetic vortices, as first argued by Greensite et.al. We show that a local gauge transformation of a certain type "creates" abelian monopole-antimonopole pairs along magnetic vortices. This gauge transformation exists in pure SU(N) gauge theory at any N.

Keywords: Solitons Monopoles and Instantons, Confinement, Lattice Gauge Field Theories.

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Understanding the mechanism of confinement in QCD has kept many peoples busy for 30 years. At present there are two basic schools of thought on the subject, both motivated in large measure by early works of 't Hooft. One school maintains that the area law of the Wilson loop, and thus confinement, comes about due to condensation of magnetic vortices in the QCD vacuum. This physical idea was proposed in the early days of QCD [1] and later formalised in [2]. This approach has been dormant for quite a while with the exception of a couple of works [3, 4]. Interest in it has been rekindled in the recent years in the lattice community following the work of Greensite and collaborators [5, 6].

The other idea is known by the name of dual superconductivity and goes back to [7]. Here one assumes that the QCD vacuum behaves in a way similar to a superconductor but with electric and magnetic quantities interchanged. In particular magnetic monopoles are supposed to be condensed in the QCD vacuum leading to the dual Meissner effect and therefore linear confinement of colour charges.

Although on the face of it the status of the two approaches is similar, in fact there is a profound difference between them. Magnetic vortices in gauge theories are physical objects in the sense that it is possible to define a gauge invariant vortex creation operator. The expectation value of this operator then can be used as a gauge invariant order parameter for confinement [2] and it indeed has been shown analytically [8] and numerically [9] that its behaviour changes above deconfinement transition. As discussed in [10] in pure Yang-Mills theories the VEV of this operator probes the mode of realisation of the global magnetic Z_2 symmetry. This symmetry is a little peculiar since its group element is not an exponential of a volume integral of some charge, but rather an exponential of a surface integral. This group element is nothing but the fundamental Wilson loop taken over the contour at spatial infinity [10]

$$M = \mathrm{Tr}\mathcal{P} \exp \left\{ ig \oint_{R\to\infty} dl_i A^i \right\}. \tag{1}$$

On the other hand no gauge invariant order parameter is known for monopoles in nonabelian theories. Moreover, although the monopoles are supposed to carry a conserved magnetic charge, no gauge invariant magnetic charge operator has ever been constructed in pure gauge theories.

Nevertheless, the monopole condensation mechanism has been studied extensively on the lattice. To define monopoles on the lattice one has to fix a particular abelian gauge. This is achieved by choosing an adjoint field $\chi = \chi^a \tau^a$ and diagonalising it by a gauge transformation. Once this diagonalisation is achieved, the residual gauge symmetry is $\mathrm{U}(1)^{N-1}$. The locations of abelian monopoles are then identified by measuring the abelian magnetic flux corresponding to the residual gauge group. The flux through an elementary plaquette is defined modulo 2π. If the total flux that emanates through all the faces of a three dimensional cube is greater than 2π, one ascribes a magnetic monopole to this particular cube. The choice of the adjoint field χ specifies the "abelian projection". It is usually taken as a component of the gauge field strength, as a Polyakov loop on a finite lattice [11], or as the lowest eigenfunction of the lattice laplacian operator [12]. Any other choice is in principle permissible. The most popular projection is the so called " maximal abelian projection" which is defined implicitly through minimisation of a particular functional of lattice gauge

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fields [13]. The detailed properties of the monopoles, such as their locations and density clearly depend on the choice of the abelian projection [11]. Nevertheless it has been found that some basic properties are common to monopoles defined in different abelian gauges, at least within certain limits [11]. For example all monopoles are condensed in the confining phase and are not condensed in the high temperature deconfined phase. Likewise the monopole density in various gauges seems to scale similarly in the continuum limit. This universality is at the root of the frequently expressed belief that abelian monopoles are *bona fide* physical objects [14] that carry a gauge invariant conserved monopole charge [15].

This interpretation is frequently taken even further. One postulates that the low energy dynamics of QCD is the same as that of a dual abelian Higgs model, where the role of the Higgs field is played by the monopole field and the role of the electric charge by the magnetic charge. The lattice data is sometimes interpreted in terms of this model [16]. The dual superconductor philosophy does not differentiate between the SU(N) gauge theories at $N = 2$ and $N > 2$. In practice this approach is most frequently applied at $N = 2$, since this theory is technically the simplest.

The purpose of this note is to show that an interpretation of this type is seriously flawed. In particular we will show that the physical Hilbert space of pure SU(2) gauge theory does not contain a physical observable which could correspond to a magnetic charge. Therefore, at least in this case, the dual Higgs model cannot be the correct low energy theory.

The argument is very simple and straightforward. Consider pure SU(2) gauge theory. Let us assume that within this theory there exists a local, gauge invariant magnetic current J^M_μ. Such a current obviously must be C-parity odd, $C^{-1}J^M_\mu C = -J^M_\mu$. This is the case in all the implementations of the dual Higgs model known to us, and quite generally must be the case as monopoles have to be charge conjugates of anti-monopoles. However, it is a basic fact that pure SU(2) gauge theory *does not possess a physical charge conjugation transformation.* This is easiest to see in the lattice formulation. The basis of physical operators of a pure SU(2) gauge theory is spanned by traces of fundamental Wilson loops of various sizes. The action of charge conjugation on any Wilson loop operator is

$$C^{-1}WC = W^* \,. \tag{2}$$

However for SU(2) the group trace of any Wilson loop is real. Therefore any physical gauge invariant observable O satisfies

$$C^{-1}OC = O \,. \tag{3}$$

Another way of seeing this, is to realise that since the two eigenvalues of W are complex conjugates of each other, the transformation (2) permutes the two eigenvalues. Thus the "would be" charge conjugation is nothing but the global Weyl subgroup of the SU(2) gauge group.

Thus there are no charge conjugation odd observables in the pure SU(2) gauge theory, and therefore magnetic current and magnetic charge do not exist. The U(1) dual abelian Higgs model therefore can not be a relevant low energy description of physical degrees of freedom.

The preceding argument leaves the door open to the existence of a Z_2 subgroup of the monopole charge group. The point is that even though a U(1) charge is naturally C-odd, there is one element of the group generated by it which is C-even. This group element $T = \exp\{\pi \int d^3x J_0^M\}$ could in principle exist[1] even if the charge itself does not. If such a Z_2 symmetry existed one could hope that the effective low energy theory could be a Z_2 gauge Higgs model. However, this too can not be the case. A field that is odd under T would be a source of magnetic flux equal to half that of the minimal Dirac string. Thus a Z_2 magnetic vortex could end on a particle created by such a field. However it is well known that only closed Z_2 vortices exist in the SU(2) gauge theory [10]. If open vortices existed, the magnetic symmetry group element M of (1) would not be conserved. We thus conclude that even if the low energy effective Z_2 gauge theory existed, it could not contain matter fields.[2]

We note that our argument applies only to pure gauge theory and does not contradict the well established fact that in the Georgi Glashow model (SU(2) gauge theory with an adjoint Higgs field) one can perfectly well define a gauge invariant magnetic charge. In this case the definition, due to 't Hooft, is

$$J_\mu^M = \partial_\nu \tilde{F}^{\mu\nu} \tag{4}$$

with

$$F^{\mu\nu} = \hat{\Phi}^a F^a_{\mu\nu} - \frac{1}{g} f^{abc} \hat{\Phi}^a (D_\mu \hat{\Phi})^b (D_\nu \hat{\Phi})^c \,, \qquad \hat{\Phi}^a = \frac{\Phi^a}{|\Phi|} \,. \tag{5}$$

where Φ^a is the adjoint Higgs field appearing in the lagrangian of the Georgi Glashow model. The charge conjugation transformation in the Georgi Glashow model is defined as

$$\begin{aligned} A_\mu^{1,3} &\to -A_\mu^{1,3} \,, \qquad A_\mu^2 \to A_\mu^2 \,, \\ \Phi^{1,3} &\to \Phi^{1,3} \,, \qquad \Phi^2 \to -\Phi^2 \,. \end{aligned} \tag{6}$$

This is clearly distinct from a global gauge transformation, since the vector potential and the Higgs field transform differently. However the transformation on the gauge potential alone is indeed equivalent to a global gauge transformation. Thus when an independent Higgs field is not present, the charge conjugation also disappears. Physically this means that as the Higgs field is made heavier, all the states in the Hilbert space which are odd under the charge conjugation also become heavy. In the limit of pure gluodynamics, or infinite Higgs mass, they become infinitely heavy and therefore unphysical.

It is instructive to see how our argument coexists with the construction of a formally gauge invariant magnetic current given in [11]. The expression suggested in [11] is the same as eqs. (4), (5) with a composite field χ^a substituted for the Higgs field Φ^a. The field χ is chosen as some operator in the adjoint representation of the gauge group in gluodynamics and is the same field which defines the abelian projection. The main problem with this expression, from our point of view, is that it is not a Lorentz vector current, since the "Higgs" which it is constructed from is not a scalar field (we remind the reader that χ is usually chosen either as some fixed component of the gauge field strength, or as a Polyakov

[1] Note that such an operator is *not* the same as the Z_2 magnetic symmetry operator M discussed in [10].

[2] We note that such an effective theory has in fact been suggested in [17], but it has nothing whatsoever to do with the dual abelian Higgs model.

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loop on the finite lattice, or in some rather implicit way). Thus the spatial integral of the zeroth component of this "current" is not a scalar charge. It is indeed clear that this expression is charge conjugation even, since the transformation properties of any adjoint χ under the "charge conjugation" (2) are the same as those of $F^a_{\mu\nu}$. Choosing χ as a Lorentz scalar is not possible in SU(2). For example, the obvious choice $\chi^a = d^{abc} F^b_{\mu\nu} F^c_{\mu\nu}$ does not exist due to the vanishing of the d-tensor in SU(2). Thus the expression constructed in [11] although gauge invariant, is not a conserved scalar charge.

Another expression was suggested in [15]. It is similar to the one in [11], except the choice of χ is somewhat more intricate. This suggestion has an additional problem, namely that the abelian gauge in [15] is only fixed up to a discrete Z_2 gauge transformation - the Weyl subgroup of the gauge group. The sign ambiguity in the definition of the magnetic charge noted in [15] is precisely the local version of the "charge conjugation" gauge transformation which is central to our argument. The incomplete gauge fixing should, strictly speaking, force the charge to vanish in any finite region of space when averaged over a gauge invariant ensemble [3].

Another definition of magnetic charge is used in the studies in the maximal abelian gauge. This gauge is defined by the requirement that the following expression (we use here continuum notations for simplicity) be globally minimized:

$$\int d^4x \left[\left(A^1_\mu\right)^2 + \left(A^2_\mu\right)^2 \right] . \tag{7}$$

The magnetic monopole current is then defined as the divergence of the third component of the dual field strength,

$$j^{MAG}_\mu = \partial_\nu \tilde{F}^3_{\mu\nu} . \tag{8}$$

There is however the following obstacle to this procedure. The MAG condition is itself invariant under the discrete subgroup of the gauge group that is the central point of our discussion. It is clearly invariant under the change of sign of A^1 and A^3, which is generated by the transformation eq. (2). Since this part of the gauge group is not fixed, in any properly generated ensemble of field configurations, for any given configuration one must find a gauge copy which differs only by this transformation. The definition of magnetic current in eq. (8) is odd under the transformation in question. Thus when averaged over the gauge copies, the current thus defined vanishes. This is another way of saying that this current is not gauge invariant.

Our proof of nonexistence of magnetic charge seems to lead to an apparent paradox. As we have mentioned above, lattice simulations of the pure SU(2) gluodynamics do find abelian monopoles. The density of these monopoles is gauge dependent, but it nevertheless seems to scale correctly in the continuum limit. How can this be the case if the monopoles themselves are not physical objects? We believe that the correct answer to this question is the one put forward in [6]. According to [6], the abelian magnetic monopoles that are found

[3] An explicit attempt at fixing the Weyl subgroup of the gauge group was made in the lattice implementation in [18]. It leads to a nontrivial problem of finding a global minimum of a certain Ising-type model.

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in lattice simulations overwhelmingly reside on magnetic vortices. The magnetic vortices are in fact physical objects and, being condensed in the vacuum, their density must scale in the continuum limit.[4] The positions of monopoles along the vortex are determined by the specific abelian projection and are more or less random. Thus although the monopoles are gauge artifacts themselves, their locations trace the locations of physical objects — magnetic vortices — and therefore scale in the continuum limit. The numerical evidence presented in [6] is very supportive of this point of view. Not only were monopoles found to reside on vortices, but also the distribution of the energy density around a monopole was found to be very similar to that around any other point on the vortex. In fact in a particular (center laplacian) gauge it was proven in [19] that monopoles are *always* located on vortices.

In the rest of this note we want to demonstrate that there is a clear connection between the local version of the gauge transformation (2), on which our argument hinges, and the fact that monopoles are located along vortices. Consider a lattice SU(2) pure gauge theory in an abelian gauge specified by diagonalisation of some adjoint operator $\chi(x)$ defined on lattice sites. In this abelian projection one defines vortices and monopoles in terms of a magnetic flux in the third direction in colour space. The abelian part P_A of every SU(2) plaquette matrix P is then defined through the Euler angle decomposition [11]

$$\begin{aligned} P(x) &= \exp\{i\alpha(x)\sigma_3\}\exp\{i\beta(x)\sigma_2\}\exp\{i\gamma(x)\sigma_3\} \\ &= \exp\{i\alpha(x)\sigma_3\}\exp\{i\beta(x)\sigma_2\}\exp\{-i\alpha(x)\sigma_3\}P_A(x)\,, \end{aligned} \tag{9}$$

with

$$P_A(x) = \exp\{i(\gamma(x) - \alpha(x))\sigma_3\} \equiv \exp\{iF(x)\sigma_3\} \tag{10}$$

where σ_i are the Pauli matrices. Here the plaquette variable is defined in a standard way as a product of the SU(2) link matrices along the plaquette:

$$P^{ab}_{\mu\nu}(x) = [U(x, x+\mu)U(x+\mu, x+\mu+\nu)U(x+\mu+\nu, x+\nu)U(x+\nu, x)]^{ab}\,. \tag{11}$$

Let us now consider a local version of the gauge transformation (2), which permutes the eigenvalues of the matrix $\chi(x)$ at one particular site X. This transformation is affected by the matrix $c(x)$ which is unity on all sites except for X, and on this site is given by σ_2. Under this transformation the plaquette matrix transforms as

$$P_{\mu\nu}(x) = c^{-1}(x)P_{\mu\nu}(x)c(x)\,. \tag{12}$$

[4]One has to be cautious with the interpretation of this statement in the following sense. While the density of physical, gauge invariant magnetic Z_2 vortices must scale correctly in the continuum limit, the existing lattice algorithms for identifying magnetic vortices are themselves not gauge invariant. Accordingly in different gauges their density has different scaling properties. For example, in the maximum center gauge the density scales [20] while in the laplacian center gauge it does not [21]. The relevant question is how well a given "vortex finding algorithm" identifies physical vortices. Indirect tests performed in [5] suggest that the maximal center gauge does the job rather well. At any rate, since the explicit expression for the conserved Z_2 charge carried by the vortices exists [10], one should be able to set up an explicitly gauge invariant procedure for identifying physical vortices on the lattice. This would involve measuring the sign of Wilson loops of various sizes, and identifying the underlying vortex structure on the basis of this data. Such a procedure however may turn out to be tricky since it may be affected by ultraviolet lattice artifacts. We thank Philippe de Forcrand for raising this issue with us.

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Thus the only plaquettes that are affected by this transformation are the ones that are associated with the site X. For all these plaquettes (i.e., all orientations $\mu\nu$) the appropriate abelian parts are conjugated by this transformation

$$\begin{aligned} c^{-1} P_{A,\mu\nu}(X) c &= P^*_{A,\mu\nu}(X)\,, \\ c^{-1} F_{\mu\nu}(X) c &= -F_{\mu\nu}(X)\,. \end{aligned} \tag{13}$$

Now, if the abelian field $F_{\mu\nu}(X)$ is small this transformation has no particular significance. However if the gauge field configuration is such that a Z_2 magnetic vortex is piercing one of the plaquettes associated with the point X, the abelian magnetic field on this plaquette is close to π. The gauge transformation then transforms the flux on this particular plaquette from π to $-\pi$ thus reversing the direction of the magnetic flux of the vortex on this particular plaquette. This of course appears equivalent to placing a monopole-antimonopole pair in the three dimensional cubes separated by this plaquette. Clearly one can perform a gauge transformation not on one site but on several sites along the magnetic vortex. A transformation of this type will place on the vortex monopole-antimonopole pairs separated by arbitrary distances. We stress again that plaquettes not pierced by a magnetic vortex do not undergo any major change as a result of this transformation.

We have thus established that the local version of the gauge transformation (2) places monopole-antimonopole pairs along Z_2 magnetic vortices. In a sense, it transforms a segment of a vortex into an antivortex. Since charge conjugation is not a physical symmetry in this system, a vortex and an antivortex are physically equivalent and thus any monopole-antimonopole pair that distinguishes between them is a pure gauge artifact. This goes nicely together with the finding of [6] that in terms of energy density monopoles are hardly at all distinguishable from other points along vortices.

Our proof of nonexistence of the magnetic charge applies strictly speaking only to the SU(2) theory. Gauge theories with higher SU(N) gauge groups do possess a physical charge conjugation symmetry and thus in those theories a conserved magnetic current can be constructed. In particular the simplest candidate for such a current would be eqs. (4), (5) with the Higgs field chosen as a bona fide scalar $\chi^a = d^{abc} F^b_{\mu\nu} F^c_{\mu\nu}$. However, even in this case, the naive dual abelian Higgs model is not a viable low energy theory, since it is supposed to involve $N-1$ magnetic charges. Some of these currents are clearly unphysical. In particular the analogue of the local gauge transformation that artificially creates monopole-antimonopole pairs clearly exists in any pure SU(N) gauge theory. In this case such a segment of a Z_N magnetic vortex can be transformed into $N-1$ antivortices, and the two are physically equivalent for any N. Thus at least some monopoles in the lattice simulations of SU(3) gauge theories are also gauge artifacts. There are also other configurations in the SU(3) gauge theory which are identified as monopoles by the existing "monopole finding" algorithms. In particular one can imagine three Z_3 magnetic vortices coming together at the same point X. Such a configuration clearly is not equivalent to a single vortex, but rather to two vortices which run together along the same line and, at the point X, split into two. These configurations must exist in lattice simulations. The question is quantitative: whether their density can be accounted for simply by the probability of two independent vortices running along the same line, or is this density significantly enhanced? Of course even these configurations do not look like monopoles with a Coulomb-like magnetic field. Whether there are genuine physical Coulomb-like monopoles in this case is an interesting question. It seems unlikely to us that this is the case, but the question is certainly well worth studying numerically. In particular, it would be interesting to extend the numerical analysis of [6] to the SU(3) theory and see if a statistically significant proportion of monopoles exists for which the fluxes through each phase of the cube are different from an integer multiple of $2\pi/3$ modulo small fluctuations. It would also be interesting to choose the abelian projection with respect to the local scalar field $\chi^a = d^{abc}F^b_{\mu\nu}F^c_{\mu\nu}$ and investigate numerically whether properties of monopoles in this particular gauge are any different from other gauges.

Note added. The first version of this note was followed by the appearance of [22]. This paper summarises the view of its authors on the nature of abelian monopoles. It disagrees with our views on two main points. First it suggests that the absence of C-parity is not an obstacle for definition of the monopole current in maximal abelian gauge. We have added a paragraph in the present version (around eqs. (7), (8)) which shows that this is not the case, and that the current defined as in [22] vanishes when averaged over gauge copies.

The second important statement made in [22] is that monopoles can not be gauge artifacts since the action measured on the plaquettes bounding the monopole is greater (in lattice units) than the average plaquette action on the lattice. In our view however this fact alone does not by itself preclude the monopoles from being gauge fixing artifacts. As we have explained, we view monopoles as points on a magnetic vortex. The identification of points on a vortex as monopoles depends on the gauge fixing and is thus a gauge fixing artifact. On the other hand, a magnetic vortex being a physical object, must carry an excess of energy (or euclidean action) at each of its points relative to the average point on a lattice. Thus it is only natural that the points on the vortex that are identified as monopoles by a particular lattice "monopole finding algorithm" also carry an excess of action. We do not claim that a monopole can be created by a gauge transformation at a point on the lattice which has a small value of the field strength and thus our argument does not imply that they should be indistinguishable from an "average" point on the lattice.

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